Semantics and Probability

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Introduction

Semantics: Relationship between language and the world

Assertions make claims about the way things are:

(1)  
   a. Peter is 60 years old!
   b. Peter is likely to retire within a decade.

Focus: Semantics of assertions about things that are uncertain.
**Goal:** Develop a compositional semantics for expressions that refer to probability and possibility

(2)  
   a. There is a 22.2% *chance* of winning in craps on one roll  
   b. The rapid strep throat test is *98% likely* to be correct.

(3)  
   a. There is a *reasonable chance* that you will win at craps.  
   b. The test is *nearly certain* to be correct.  
   c. The *likelihood* of swine flu reaching Colorado is *high*.

Probability and Possibility Expressions (PPEs):

- *chance*, *probable*, *possibility*, *likelihood certain(ly)*, *chance*, *definite(ly)*, *doubtful(ly)*, *impossible*, *likely*, *necessary*, *sure*, *uncertain*, *unlikely*

- Non-verbal expressions (adverbs, adjectives, nouns)
- Modal expressions (typically take propositional complements)
- *Gradable* predicates
PPEs in the “Real” World

PPEs have been discussed extensively in military-intelligence, meteorological, medical, and business contexts (Johnson 1973; Wallsten, Budescu, Rapoport, Zwick, and Forsyth 1986; Capriotti and Waldrup 2005; Cohn, Cortés, Vázquez, and Alvarez 2009)

- Also known as Vague Probability Expressions, Qualitative Expressions of Uncertainty, Verbal Expressions of Uncertainty and Estimated,
- Assumption: Interpreted as denoting some part of \([0, 1]\) interval of mathematical probability.
- Goal: Provide “objective” standard for vague verbal expressions - prescriptive and descriptive

<table>
<thead>
<tr>
<th>Weather reporting standards (NOAA)</th>
<th>20%</th>
<th>Slight Chance of Showers</th>
</tr>
</thead>
<tbody>
<tr>
<td>30%, 40%, 50%</td>
<td>Chance of Showers</td>
<td></td>
</tr>
<tr>
<td>60%, 70%</td>
<td>Likely Showers</td>
<td></td>
</tr>
<tr>
<td>80%, 90%, 100%</td>
<td>Showers</td>
<td></td>
</tr>
</tbody>
</table>
Empirical Studies of PPEs

Early Army research (Johnson 1973) used a simple paradigm

*This is a study to determine the meaning of some common words for certainty, in the booklets you’ve received, you will find pairs of sentences like the following set:*

- The official weather forecast states that rain is somewhat likely tomorrow.
- This means there are —— chances out of 100 of rain tomorrow.

*In the second sentence you should place a number from 0 to 100 describing the degree of certainty you think the sentence indicates.*

**Results:**

<table>
<thead>
<tr>
<th>Gradable Predicate</th>
<th>Mean</th>
<th>Std. Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>highly probable</td>
<td>82.0</td>
<td>14.3</td>
</tr>
<tr>
<td>very probably</td>
<td>78.8</td>
<td>15.7</td>
</tr>
<tr>
<td>very likely</td>
<td>73.8</td>
<td>19.2</td>
</tr>
<tr>
<td>quite likely</td>
<td>68.5</td>
<td>18.9</td>
</tr>
<tr>
<td>likely</td>
<td>60.9</td>
<td>18.5</td>
</tr>
<tr>
<td>probable</td>
<td>61.5</td>
<td>18.0</td>
</tr>
<tr>
<td>fairly likely</td>
<td>54.1</td>
<td>21.3</td>
</tr>
<tr>
<td>possible</td>
<td>50.6</td>
<td>16.9</td>
</tr>
<tr>
<td>fair chance</td>
<td>48.9</td>
<td>20.7</td>
</tr>
<tr>
<td>unlikely</td>
<td>22.9</td>
<td>15.5</td>
</tr>
<tr>
<td>fairly unlikely</td>
<td>21.3</td>
<td>14.9</td>
</tr>
<tr>
<td>improbable</td>
<td>16.3</td>
<td>15.3</td>
</tr>
<tr>
<td>very unlikely</td>
<td>14.9</td>
<td>12.5</td>
</tr>
<tr>
<td>quite unlikely</td>
<td>14.4</td>
<td>12.6</td>
</tr>
<tr>
<td>highly improbable</td>
<td>12.6</td>
<td>17.7</td>
</tr>
</tbody>
</table>
Empirical Investigation of PPEs

Kipper and Jameson (1994) investigated modal adverbs (and verbs) in German using a “wheel of fortune” methodology of (Wallsten, Budescu, Rapoport, Zwick, and Forsyth 1986)

In this game, one of eleven wheels of fortune is spun. The wheels differ widely in the sizes of their black and white portions. A player wins if the arrow to the right of the wheel points into the black sector when the wheel stops. . . . Given a particular wheel and a particular adverb phrase, the subjects were to indicate how “realistic” they judged this phrase to be . . . .

Ich habe vermutlich gewonnen
Results from Kippers and Jameson

Adverb Phrases

- auf jeden Fall (in any case)
- sicher (surely)
- gewiß (doubtless)
- bestimmt (certainly)
- höchstwahrscheinlich (very probably)
- wahrscheinlich (probably)
- wohl (I suppose)
- vermutlich (presumably)
- möglicherweise (possibly)
- vielleicht (maybe)
- eventuell (perhaps)
- auf keinen Fall (no way)
Linguistic Semantic Analyses of PPEs

Classical Modal analysis (Hintikka 1969; Kratzer 1977; Kratzer 1981): PPEs (like other modals) are *implicit quantifier* over accessible possible worlds

(4) a. It is possible that Peter will retire.
   b. \( \exists w \, \text{Acc}(w_c,w) \) [Peter retires in w]

Ignores grades of modality

Kratzer (1981) uses ordering semantics for this, e.g.:

(5) **Necessity**: \( p \) is a human necessity with respect to a modal base \( mb \) and an ordering source \( os \) iff \( \forall w \, [w \in mb \land \neg \exists w' \leq os w \rightarrow [w \in p] \)

(6) **Slight possibility**: \( p \) is a slight possibility with respect to a modal base \( mb \) and an ordering source \( os \) iff:
   i. \( \exists w \, [w \in p \land w \in mb] \), and
   ii. \( \neg p \) is a necessity in \( w \) with respect to \( mb \) and \( os \)

PPE semantics explicated in terms of these grades:

(7) a. It is slightly possible that it will rain.
   b. **it will rain** is a slight possibility

Problem: Not compositional (*very slightly possible*, *extremely unlikely*, *nearly certain*, . . . )
Gradable Predicates

Gradable predicates take degree modifiers and specifiers and appear in the comparative:

(8)  a. John is quite tall.
    b. This is 60-page long book.
    c. Terry is more athletic then Joe is. Compare: Non-gradables

(9)  a. ??Fifi is very female.
    b. ??Fifi as two chromosome female.
    c. ??Fifi is more female than Fido.

PPEs are like other gradable predicates

(10)  a. It is quite likely that it will rain.
    b. There’s a 60 % probability that she will be late.
    c. It is more probable that it will rain than that it will snow.

Proposal: Provide PPEs with a degree-based semantics.
Semantics of Gradable Predicates

Gradable predicates - relations between individual and degree on scale (Klein 1980; Cresswell 1977; von Stechow 1984; Kennedy 1999) and a standard of comparison:

**Scale**  Ordered set of degrees (values on some dimension) associated with predicate

**Standard**  Degree used in simple positive cases to distinguish those in extension of predicate from those not

(11)  

a. John is tall.  
b. $\exists d \ [\text{tallness}(\text{John}) = d \land d_{\text{tall}} \leq d]$

Simple positive degree predication decomposed into relation and null positive morpheme (existential closure of degree argument)

(12)  

a. $[\text{tall}] = \lambda x, d \ [\text{tallness}(x) = d]$  
b. $[\text{pos}] = \lambda P \ \lambda x \ \exists d \ [P(d,x) \land d_{P} \leq d]$
Degree modifiers operate on standard of comparison:

**Shifting it up:**

(13) a. John is very tall.
    b. $\exists d \ [\text{tallness}(John) = d \land \text{high}(d, d_{\text{tall}})]$

**Specifying the exact degree**

(14) a. John is six feet tall.
    b. $\exists d \ [\text{tallness}(John) = d \land 6\text{ft} \leq d]$

**Or comparing it to another degree:**

(15) a. John is taller than Mary.
    b. $\exists d [\text{tallness}(John) = d \land \exists d' [\text{tallness}(Mary) = d' \land d > d']]$
Classification of Gradable Predicates

Kennedy and McNelly (2005) classification on basis of *scale* and *standard*

**Scales:** *open* vs. *closed*

Felicity of *completely* diagnostic of *open/closed* contrast:

(16)  a. *The man is completely tall.

    b. The paint is completely dry.

    c. The door is completely open/closed.

Open-scale expressions: *tall, rich, far*

Close-scale expressions: *dry, healed, near*

Note: Scales can also be *positive* or *negative:*

(17)  a. ??John is six feet short.

    b. ??Ted is taller than Maria is short.

(18)  a. John is six inches taller than Maria.

    b. Maria is six inches shorter than John.
Negative scales often formed from positive scales on same dimension

\[ S_{\text{tall}} = \langle D_{\text{heights}}, \leq \rangle \]
\[ S_{\text{short}} = \langle D_{\text{heights}}, \geq \rangle \]

Explanation of degree-specifier effect involves treating degrees as intervals:

(19)  
\begin{align*}
\text{a.} & \quad \text{John is six feet tall.} \\
\text{b.} & \quad \exists d \ [\text{tallness(John)} \geq d \land 6\text{ft} \leq d]
\end{align*}

(20)  
\begin{align*}
\text{a.} & \quad *\text{John is six feet short} \\
\text{b.} & \quad \exists d \ [\text{tallness(John)} \leq d \land 6\text{ft} \leq d]
\end{align*}
Classification of Gradables

**Standards of Comparison** can be: contextual, absolute (minimal) or absolute (maximal)

Contextual standards:

(21)  
   a. The jockey is tall.  
   b. The goalie is tall.

Absolute standards:

(22)  
   a. The socks are damp.  
   b. The road is flat.

Comparative uses are diagnostic:

(23)  
   a. Mary is taller than John is  
   b. The lawn is damper than the porch.  
   c. The ice sheet is flatter than the road surface.
Questions about Gradable Predicates

- What kind of scale structure does it have?
  - closed, open
  - negative, positive
- What kind of standard does it have
  - Contextual
  - Absolute (minimal/maximal)
Clearly PPEs are gradable predicates:

(24)  
   a. It is quite likely that it will rain.  
   b. There’s a 60% probability that she’ll be late.  
   c. It’s more likely that it will rain than that it will snow.

Questions for a degree semantics of PPEs:

• What are the scales associated with PPEs?
• How are the degrees on the scale measured and compared?
• Are the scales for probability, possibility, likelihood etc. the same?
• What are the standards associated with PPEs like (contextual, absolute)?
Any degree semantics for PPEs will provide an analysis like this:

(25)  

a. It is probable that Federer will win.

b. \( \exists d \ [\text{probability}(\text{Federer-wins}) = d & d_{\text{probable}} \leq d] \)

Questions concern nature the scale and the standard of comparison. One thing seems certain: degrees in scale are additive and have the following properties:

- if \( p \) is a tautology \( \text{probability}(p) = 1 \),
- if \( p \) is a contradiction then \( \text{probability}(p)=0 \)
- \( \text{probability}(p) + \text{probability}(q) = \text{probability}(p \text{ or } q) - \text{probability}(p \text{ and } q) \)

But what are the degrees for PPEs (what is the analog of \textit{height}?)

Two potential answers:

- PPE scales are constructed out of propositions and orderings
- PPE scales are constructed out of mathematical probabilities (\( \langle[0, 1], \leq\rangle \))
Kratzer-style Possible-worlds Based Degree Analysis

Portner (2009) uses Kratzer’s (1981) notion of better possibility as basis for semantics:

(26)  
a.  $p$ is a **better possibility** than $q$ iff, for every accessible $q$-world, there is an accessible $p$-world which is as least as close to the ideal defined by the ordering source, but not vice versa.

b.  *A is more likely than B* is true in world $w$, with respect to an ordering source $os$ iff $A$ is a better possibility than $B$ in $w$ with respect to $os$.

Degrees are equivalence classes of propositions under the better possibility ordering, and scales are defined as follows:

(27)  
a.  $S_{prob} = \{ P : \exists p[p \in P \land \forall q[\forall r[p \text{ is a better possibility than } r \rightarrow q \text{ is a better possibility than } r] \rightarrow q \in P] \}\}$

b.  $\leq_{prob} = \{ \langle P, Q \rangle : \forall p \in P \forall q \in Q[q \text{ is a better possibility than } p] \}$
Possible-worlds Based Analysis Illustrated

On this approach degrees are sets of equi-probable propositions (where the ordering of propositions in terms of likelihood is given by a contextually salient ordering source (which also induces the *better possibility* ordering for this scale)

Standards of comparison are degrees (sets of propositions)

(28) a. It is more likely to rain than to snow for every world in which it snows there is an (accessible) world in which it rains which is more highly ranked

(29) a. It is likely to rain
b. for every world in which it snows there is an (accessible) world in which it rains which is more highly ranked then a contextually given set of propositions that count as the minimum *likely* set of propositions.

Problems: Where does the ordering come from? What is 30% *likely* on this approach?
Outline of a Probability-based Degree Semantics

Alternative Natural analysis: Scale: $[0, 1]$ interval with $\leq$ as the ordering; shared by *likely, possible, probable*, etc.

(30)  
  a. It is 30% likely that Federer will win
  b. $\exists d \ [\text{probability}(\text{Federer-wins}) = d \land 0.3 \leq d]$

(31)  
  a. It is very likely that Federer will win
  b. $\exists d \ [\text{probability}(\text{Federer-wins}) = d \land \text{high}(d, [0, 1])]$

(32)  
  a. It’s more likely that Federer will win than that Herberger will.
  b. $\exists d \ [\text{likelihood}(\text{Federer-wins}) = d \land \exists d' \ [\text{likelihood}(\text{Herberger-wins}) = d' \land d > d']]$
Natural intuitive treatment:

- *likely* and *probably* have contextual standards (like *tall*)
  
  \[(33) \quad a. \text{ It is likely that Federer will win.} \]
  \[b. \exists d \ [\text{probability(Federer-wins)} = d \amp d_{\text{likely}} \leq d \]  

- *possible*, *certain* have *absolute* standards (*minimal* and *maximal*, respectively, like *wet* and *dry*)
  
  \[(34) \quad a. \text{ It is possible that Federer will win.} \]
  \[b. \exists d \ [\text{probability(Federer-wins)} = d \amp \text{Min}([0,1]) < d \]  
  
  \[(35) \quad a. \text{ It is certain that Federer will win.} \]
  \[b. \exists d \ [\text{probability(Federer-wins)} = d \amp \text{Max}([0,1]) = d \]
Simple Naive PPE Classification

Positive vs. Negative degrees: Negative PPEs work like other negatives-scale gradables

(36)  
  a. It is 30% likely that it will rain.
  b. *It is 10% doubtful/unlikely that it will rain.

(37)  
  a. It is 30% likelier that it will rain than that it will snow.
  b. It is 30% more doubtful/unlikely that it will snow than that it will rain.

Intuitive Classification:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Standard</th>
<th>Polarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>likely</td>
<td>contextual</td>
<td>positive</td>
</tr>
<tr>
<td>unlikely</td>
<td>contextual</td>
<td>negative</td>
</tr>
<tr>
<td>possible</td>
<td>minimal</td>
<td>positive</td>
</tr>
<tr>
<td>impossible</td>
<td>maximal</td>
<td>negative</td>
</tr>
<tr>
<td>certain</td>
<td>maximal</td>
<td>positive</td>
</tr>
<tr>
<td>uncertain</td>
<td>maximal</td>
<td>negative</td>
</tr>
</tbody>
</table>
Empirical Challenges for a PPE Degree Semantics

Variability of modifiers (particularly in nominal domain)

(38)  

a. There is a 60%/high/larger/?large/strong/?good/?better probability that Federer will win.

b. There is a *60%/high/*large/*larger/strong/good/better possibility that Federer will win.

c. There is a 60%/high/?large/?larger/strong/good/better chance that Federer will win.

Also lexical variation:

(39)  

a. *This is 20% probable.

b. This is 20% likely.

And cross-linguistic variation:

(40)  

a. Es ist gut/*ganz möglich, daß er die Zeitung gelesen hat.

b. It is completely/*good possible that he has read the newspaper.
Kennedy notes: Typically contextual standards go with OPEN scale predicates, but *probable* and *likely* are contextual and [0,1] is clearly closed.

(41) a. *It is completely probable that it will rain.
   b. *It is completely likely that it will rain.

*Completely* should force gradable to have absolute maximal interpretation:

(42) a. The glass was filled.
   b. The glass was completely filled.

And: *completely possible* \(\neq\) *certain!*

(43) It is completely possible that it will rain.
Pragmatic Scales for *Possible* and *Certain*

Are absolute PPEs (*possible* or *necessary*) gradable predicates at all? (difficult question: is *flat* gradable (Lasersohn 1999)?)

(44) a. It is completely possible that class will go well on Monday.
b. It is very possible that class will go well Monday.

(45) a. It is completely necessary that class go well on Monday.
b. It is very necessary that class go well Monday.

Comparative:

(46) a. ?It is more necessary that Federer will win than that Herburger will.
b. ?It is more possible that Federer will win than that Herburger will.

(47) a. This road is very flat.
b. This road is flatter than that one.

(48) a. This woman is very pregnant
b. ?This woman is more pregnant than that one.
Observations on *Completely Possible*

Completion modifiers can have minimal reading: *not impossible* or heightened reading *possibility to be reckoned with*)

(49)  
   a. It is completely possible that if you flip 10 coins all of them will come up heads.  
   b. It is entirely possible that we will run into him here.

(50)  
   a. It is completely necessary that you turn in those grades.  
   b. It is entirely unnecessary that you

Perhaps a speech act operator *I am completely sure that*...?
Future directions

- Develop a semantic account founded on a modal-semantic account of probability scales
- Annotate PPEs in corpora:
  - Identifying PPEs and their modifiers (and uses)
  - Provide a normalized-scale interpretation for PPEs in context

(51) It is <PROBEX pid='pe1' prob = .6> likely </PROBEX> to rain.

- Identify relational information among PPEs

(52) The House is <PROBEX pid='pe1' prob = .6> likely </PROBEX> next week to take up a Bush administration proposal to empower the Treasury to back up embattled mortgage giants Fannie Mae and Freddie Mac. Lawmakers will <PROBEX pid='pe2' relprobex='pe1' prob = .8> probably </PROBEX> accommodate the broad outlines of a proposal by Treasury Secretary Henry Paulson to offer explicit backing for the two government-sponsored enterprises.

(Jeanne Sahadi, CNNMoney.com, July 18, 2008)
Conclusion

- Semantic theory as a way of understanding how we talk about probability (i.e. use PPEs)
  M. Lieberman *LanguageLog*: English speakers speak of probability much the way Piranha do of numbers
- PPEs present challenges in terms of determining appropriate scales and standards
- Intuitive [0,1] probability scale doesn’t seem to work quite right, but almost
- Empirical facts may trump theory (as soon as we understand them)
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Semantics and Probability

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