Switching in Self-Controlled Chaotic Neuromodules *

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Abstract

The activity dynamics of recurrent neural networks can exhibit deterministic chaos due to the nonlinear transfer functions. Chaotic attractors are winded around infinitely many unstable periodic orbits. Each can be stabilized by a feedback control. In the present article we explore the flexibility of the chaotic dynamics of a recurrent neuromodule and construct a neural controller which is able to switch between several periodic patterns in two different ways: either deterministically by external inputs or spontaneously by dynamic noise. The chaotic attractor acts as an intermediate state between successively stabilized dynamic patterns. It has all the possible motions present, like an attentive state between different stimuli.

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1 Introduction

The important role of deterministically chaotic dynamics for all fields of the neurosciences has become clear in recent years. Not only observations of bifurcation sequences and chaotic attractors in biological systems but also the functional aspects for computational purposes are discussed extensively in the literature. Only a few highlights selected on personal taste can be mentioned here.

Besides the classsical paper by Skarda & Freeman (1987), summarizing the discussion up to the mid of the past decenium, there are signs of chaos reported on the different levels in the neurobiological sciences (see e.g. Freeman, 1992, and recently Hayashi & Ishizuka, 1995). Functional aspects for describing principles of biological systems as well as artificial intelligent agents are discussed by many authors, impressively e.g. in Babloyantz & Lourenço (1994). In that paper especially controlling chaos is used for feature detection where the controlled spatio-temporal orbits act as an analyzer.

Infinitely many periodic orbits build a sceleton in the nonperiodic chaotic attractor determining its basic topological structure (Procaccia, 1987). The basic idea of controlling chaos is to stabilize one of the infinitely many periodic orbits by a feedback control (Ott, Grebogi, Yorke, 1990). This mechanism only needs small control signals since the neighborhood of any present period is visited automatically after a transient time due to the chaotic dynamics.

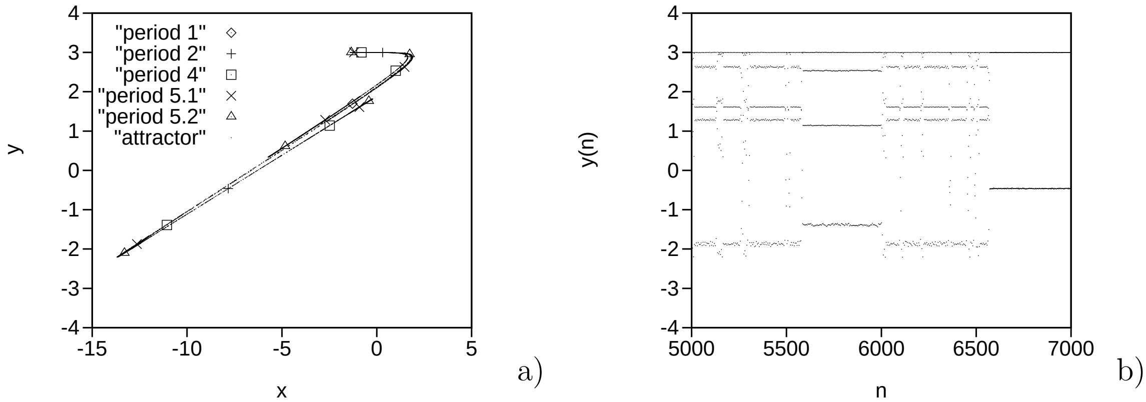

Figure 1: *In a) periodic points of period 1, 2, 4 and of two periods 5 are shown in their position along the attractor. No period 3 exists in the system under study. In b) the time series of spontaneous switching by internal dynamic noise between three controlled periods 5.1, 4 and finally 2 is shown. In between randomly occuring escapes to transients along the chaotic attractor are visible. The chaotic attractor acts as a search state between the unstable periodic orbits.*

One of the first articles proposing chaos control for information processing dates back to 1991 (Ding & Kelso). However, in all preceeding applications either the controller or the chaotic system itself (e.g. Alsing *et al.*, 1994) is purely algorithmic. We propose a self-controlled chaotic neuromodule in which the chaotic submodule (a time discrete activity dynamics of two neurons with sigmoidal transfer function, Pasemann 1995) as well as the chaos controller is constructed by the same type of neurons in a consistent neural structure (Stollenwerk & Pasemann, 1996(b)). Therefor a simple feedback controller is implemented using least squares control (Stollenwerk & Pasemann, 1996(a)) which is based on the principles of chaos control described in Ott *et al.*, 1990, and Romeiras *et al.*, 1992, with modifications similar to Reyl *et al.*, 1993) and which copes with the additional difficulty of a delay in the feedback (Stollenwerk, 1995).

Then a very effective single point control (Stollenwerk & Pasemann, 1996(a)) can be implemented to demonstrate the phenomenon of switching between different periodic orbits for the first time in a consistent self-controlled neuromodule (Stollenwerk & Pasemann, 1996(b)). Two different mechanisms of switching can be distinguished at present: deterministic switching by external inputs and spontaneous switching by dynamic noise. In both switching mechanisms the chaotic attractor links the different periods. After switching off, respectively escaping from one controlled period the system passes along the chaotic attractor in a transient before getting captured in another controlled period. The spontaneous control may serve as an attention mechanism: Tuning internal parameters, i.e. the sizes of the controlling regions, the relative frequency for visiting the different periods can be changed. This leads to a temporary selection of specific periods which can be used for a further analysis as in the Babloyantz\Lourenço machine.

2 Consistent neural self-control

For illustration of the principles we use the simplest possible chaotic neuromodule and the simplest control algorithm available: a time discrete two neuron module $\underline{x}_n := (x_n, y_n)^{tr}$ with time step n and sigmoidal transfer function $\sigma(x) := 1/(1+e^{-x})$, controlled by a linear least squares controller $\underline{p}_n := (p_n, 0)^{tr}$. With the convention that $\sigma(\underline{x})$ denotes componentwise application of σ to $\underline{x}$, obtaining a vector of the same dimension as $\underline{x}$, we can write the dynamics of the composed system

$$\underline{x}_{n+1} = \underline{\vartheta} + W \cdot \sigma(\underline{x}_n) + \underline{p}_n \quad . \tag{1}$$

Without control the system is chaotic in wide parameter regions, e.g. for

$$\underline{\vartheta} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \quad , \quad W = \begin{pmatrix} -20 & 6 \\ -6 & 0 \end{pmatrix}$$

the chaotic attractor is shown in Fig.1a. Depicted are also the first unstable periodic orbits 1, 2, 4 and the two periods five: 5.1 and 5.2. All these orbits can be stabilized in such a way that the control is an additional dynamical variable, i.e. $\underline{p}_{n+1} = p_n(\underline{x}_n)$ taking into account the delay due to the feedback.

Now the control which algorithmically is given by linear least squares deviations between actual dynamics $\underline{x}_n$ and goal dynamics, i.e. the periodic orbits $\underline{x}_{P,i}$

can be approximated by a hidden layer of four neurons $\underline{z} = (z_1, ..., z_4)^{tr}$ for each control region. The control regions are defined by the constraint that the controlling signal be small, hence zero outside a neighborhood of the periodic point. It turned out that at least the first five periods can be stabilized by applying the control signal only to one of the periodic points. Therefor we call it *single point control*. For one period the complete self-controlling system reads

$$\begin{aligned} \underline{x}_{n+1} &= \underline{\vartheta} + W \cdot \sigma(\underline{x}_n) + V \cdot \sigma(\underline{z}_n) \\ \underline{z}_{n+1} &= \underline{\theta} + U \cdot \sigma(\underline{x}_n) \quad , \end{aligned} \tag{2}$$

with matrices V, θ and U fully determined by the control algorithm:

$$V = \begin{pmatrix} v_1 & v_2 & v_3 & v_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad , \quad \underline{\theta} = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_4 \end{pmatrix} \quad , \quad U = \begin{pmatrix} u_{11} & 0 \\ u_{21} & 0 \\ u_{31} & 0 \\ u_{41} & 0 \end{pmatrix} \quad .$$

In order to stabilize N different unstable periodic orbits we have to introduce correspondingly N different controlling regions.

Hence, with upper index $\nu = 1, ..., N$, the system can be written as

$$\begin{aligned} \underline{x}_{n+1} &= \underline{\vartheta} + W \cdot \sigma(\underline{x}_n) + \sum_{\nu=1}^{N} V^\nu \cdot \sigma(\underline{z}_n^\nu) \\ \underline{z}_{n+1}^\nu &= \underline{\theta}^\nu + U^\nu \cdot \sigma(\underline{x}_n) \quad . \end{aligned} \tag{3}$$

Here ν e.g. indicates the periods 2, 4 and 5.1, and $\underline{z}_n^\nu$ denotes the activities of the four neurons of one controller $C(\nu)$.

3 Switching between different orbits

The sequential activation of different controllers, called a *switching program*, is determined by strong external inhibiting inputs $\underline{I}_n^\nu$ during a given number of time steps. It is included in the consistent neural self-control as follows

$$\begin{aligned} \underline{x}_{n+1} &= \underline{\vartheta} + W \cdot \sigma(\underline{x}_n) + \sum_{\nu=1}^{N} V^\nu \cdot \sigma(\underline{z}_n^\nu) \\ \underline{z}_{n+1}^\nu &= \underline{\theta}^\nu + U^\nu \cdot \sigma(\underline{x}_n) + \underline{I}_n^\nu \quad . \end{aligned} \tag{4}$$

We start with a strong constant inhibition $\underline{I}_n^\nu$ for the neurons of all controllers, hence letting freely develope the chaotic dynamics. To control a specific orbit ν controller $C(\nu)$ is liberated by setting the corresponding inhibiting inputs to zero. This method we call *deterministic switching by external inputs.*

In contrast now we describe another possibility of *spontaneaous switching by dynamic noise*. When activating all three controllers at a time, i.e. all external control inputs $\underline{I}_n^\nu$ are set to zero, the system will, after a transient on the chaotic attractor, enter one control area, being controlled there forever. But if an internal or external dynamic noise term $\underline{\xi}_n$ is introduced into the chaotic module the system can escape from a once controlled orbit and move around the attractor until being eventually captured in another control area. This new periodic orbit will be destabilized again by the noise, and so on. The total system displaying this spontaneous switching reads

$$\begin{aligned} \underline{x}_{n+1} = & \; \underline{\vartheta} + W \cdot \sigma(\underline{x}_n) + \sum_{\nu=1}^{N} V^\nu \cdot \sigma(\underline{z}_n^\nu) + \underline{\xi}_n \\ \underline{z}_{n+1}^\nu = & \; \underline{\theta}^\nu + U^\nu \cdot \sigma(\underline{x}_n) \quad . \end{aligned} \tag{5}$$

The simplest type of noise, i.e. Gaussian white noise with constant variance, is sufficient to induce spontaneous switching between orbits of period two, four and five. For example in the part of the time series depicted in Fig.1b all three orbits 2, 4 and 5.1 are visited. Although each period is controlled only by single point control the period five orbit is stabilized most often, eventually destabilized by the noise and revisited after a short transient. At around time $n = 5600$, however, the period-4 orbit is met after a transition and can be stabilized for about 400 time steps before becomming unstable again. After these switches the system even locks in to the period-2 orbit at around time step 6600. As outlined above this spontaneous switching is an interesting variant for further investigations.

References

[1] Alsing, P.M., Gavrielides, A., & Kovanis, V. (1994). Using neural networks for controlling chaos. *Physical Review E*, **49**, 1225–1231.

[2] Babloyantz, A., & Lourenço, C. (1994). Computation with chaos: A paradigm for cortical activity. *Proceedings of the National Academy of Sciences USA*, **91**, 9027–9031.

[3] Ding, M., & Kelso, S. (1991). Controlling chaos: A selection mechanism for neural information processing. In D. W. Duke, W. S. Pritchard (Eds.), *Measuring Chaos in the Human Brain* (pp. 17–31). Singapore: World Scientific.

[4] Freeman, W.J. (1992). Tutorial on neurobiology: From single neurons to brain chaos. *Internationl Journal of Bifurcation and Chaos*, **2**, 451–482.

[5] Hayashi, H., & Ishizuka, S. (1995). Chaotic responses of the hippocampal CA3 region to a mossy fiber stimulation in vitro. *Brain Research*, **686**, 194–206.

[6] Ott, E., Grebogi, C., & Yorke, J.A. (1990). Controlling chaos. *Physics Review Letters*, **64**, 1196–1199.

[7] Pasemann, F. (1995). Neuromodules: A dynamical systems approach to brain modelling. In H. Herrmann, E. Pöppel, D. Wolf (Eds.), *Supercomputing in Brain Research - From Tomography to Neural Networks*, (pp. 331–347). Singapore: World Scientific.

[8] Procaccia, I. (1987). Exploring deterministic chaos via unstable periodic orbits. *Nuclear Physics* **B**, *Proceedings Supplement*, **2**, 527–538.

[9] Reyl, C., Flepp, L., Badii, R., & Brun, E. (1993). Control of NMR-laser chaos in high-dimensional embedding space. *Physics Review E*, **47**, 267–272.

[10] Romeiras, F.J., Grebogi, C., Ott, E., & Dayawansa, W.P. (1992). Controlling chaotic dynamical systems. *Physica D*, **58**, 165–192.

[11] Skarda, C.A., & Freeman W.J. (1987). How brains make chaos in order to make sense of the world. *Behavioural Brain Science*, **10**, 161–195.

[12] Stollenwerk, N. (1995). Self-controlling chaos in neuromodules. In H. Herrmann, E. Pöppel, D. Wolf (Eds.), *Supercomputing in Brain Research - From Tomography to Neural Networks*, (pp. 421–426). Singapore: World Scientific.

[13] Stollenwerk, N. & Pasemann, F. (1996(a)). Control Strategies for Chaotic Neuromodules, *International Journal of Bifurcation and Chaos*, **6**, 693–703.

[14] Stollenwerk, N. & Pasemann, F. (1996(b)). Consistent Neurocontrol of Chaotic Neuromodules, *submitted for publication.*