Attractor Switching by Neural Control of Chaotic Neurodynamics *

Frank Pasemann and Nico Stollenwerk†
Max-Planck-Institute for Mathematics in the Sciences
D-04103 Leipzig, Germany

f.pasemann@mis.mpg.de and nks22@cam.ac.uk

Abstract

Chaotic attractors of discrete-time neural networks include infinitely many unstable periodic orbits, which can be stabilized by small parameter changes in a feedback control. Here we explore the control of unstable periodic orbits in a chaotic neural network with only two neurons. Analytically a local control algorithm is derived on the basis of least squares minimization of the future deviations between actual system states and the desired orbit. This delayed control allows a consistent neural implementation, i.e. the same types of neurons are used for chaotic and controlling modules. The control signal is realized with one layer of neurons, allowing selective switching between different stabilized periodic orbits. For chaotic modules with noise random switching between different periodic orbits is observed.

*appeared in: *Network: Computation in Neural Systems*, **9**, 549–561.

†Department of Plant Sciences, University of Cambridge, Downing Street, Cambridge CB2 3EA, UK

1 Introduction

An essential feature of recurrent networks is their multi-functionality based on their inherent complex dynamical properties. This corresponds to findings in biological brain research, where oscillations of various periodicities, as well as synchronization effects, wave patterns of activity, and even chaotic dynamics are observed on different levels of signal processing. From this data it is now evident that non-linear dynamics is fundamental for understanding higher level brain functions. In particular, chaotic dynamics is frequently observed in biological neural networks, although its functional role is still obscure (Guevara, et al., 1983; Babloyantz, et al., 1985; Babloyantz and Destexhe, 1986; Freeman, 1988, 1992; Elbert, et al., 1994; Freeman and Barrie, 1994; Schiff, et al., 1994; Hayashi and Ishizuka, 1995). One hypothesis is that chaotic dynamics endows a neural system with the ability to respond rapidly and with a flexible repertoir of behaviors to a changing environment (Skarda and Freeman, 1987; Freeman, 1993).

The control of chaos in non-linear systems is a mechanism which can be used effectively for information processing (see e.g. Kapitaniak, 1996). The basic idea is to stabilize one of the infinitely many unstable periodic orbits in a chaotic attractor by feedback control (Ott, et al., 1990). The relevance of this method for neural systems has been demonstrated for instance by Ding and Kelso (1991) (following the general ideas of Freemann), Lourenço and Babloyantz (1994) (suggested for pattern recognition and motion detection), Babloyantz and Lourenço (1994) (applied to biologically oriented models), and Schiff, et al. (1994) (applied to *in vitro* experiments). The basic feature of this method is that specific oscillatory modes, which may code for instance behavior relevant stimuli, are linked by seemingly chaotic transient states. This kind of switching between different attractors is realized with less effort than crossing boundaries of corresponding basins of attraction or changing the attractor and basin structure, respectively, of the whole system.

The intention of the present work is to show that chaotic dynamics as well as its control can be realized in one and the same modular neural network. In previous publications either the chaotic system is a neural network and external control is purely algorithmic (Sepulchre and Babloyantz, 1993) or neural control is applied to algorithmic chaotic systems, like e.g. the dissipative Hénon map (Alsing, et al., 1994; Der and Herrmann, 1994). Here, the combination of a chaotic neuromodule with neural control modules is understood as a first and simple example for the interaction of functionally differentiated neural systems. For demonstrating specific effects of such an interaction, we choose the discrete chaotic dynamics of a simple two neuron network (Pasemann 1995). An appropriate control algorithm stabilizing a desired periodic orbit is then implemented into a feedforward network with one layer of neurons. To achieve this, we have to reformulate the control algorithm using least square control similar to Reyl et al. (1993), Stollenwerk and Pasemann (1996) and delayed feedback similar to

Dressler and Nitsche (1992), Stollenwerk (1995).

Basic properties of the 2-neuron module (Pasemann, 1995) are summarized in section 2, and the corresponding implementable control is derived in section 3. To allow switching between different oscillatory modes, we have to use local control of specific periodic points. A 4-neuron control layer implements the necessary cut-off function and the corresponding control function at the same time as shown in section 4. Each of the stabilized periodic orbits needs a separate control layer, and the control is activated for instance by disinhibition of the corresponding neurons. Deterministic switching between different oscillatory modes is then achieved by activating one control device at a time. Spontaneous switching between different periodic orbits appears, if all controllers are active at a time. This is demonstrated in section 5 for unstable orbits with periods two, four and five. The last section gives a discussion of the results.

2 A Minimal Chaotic Neuromodule

The system subject to control is a recurrent two neuron module with an excitatory neuron and an inhibitory neuron with self-connection (Figure 1a). Its activity at time n is given by $\underline{x}_n = (x_n, y_n)$, and the discrete time dynamics

$$\underline{x}_{n+1} = \underline{f}(\underline{x}_n) \quad , \quad \underline{f} : \mathbf{R}^2 \to \mathbf{R}^2 \quad ,$$

is defined by the map

$$\begin{aligned} x_{n+1} &= \vartheta_1 + w_{11} \cdot \sigma(x_n) + w_{12} \cdot \sigma(y_n) \quad , \\ y_{n+1} &= \vartheta_2 + w_{21} \cdot \sigma(x_n) \ , \end{aligned} \tag{1}$$

where σ denotes the sigmoidal transfer function defining the output of a neuron:

$$\sigma(x) := \frac{1}{1+e^{-x}} \quad . \tag{2}$$

This module exhibits chaotic dynamics for large parameter domains (Pasemann, 1995). In the following we will discuss properties of the global chaotic attractor existing for the parameter set

$$\begin{aligned} &\vartheta_1 = -2 \quad , \quad w_{11} = -20 \quad , \quad w_{12} = 6 \quad , \\ &\vartheta_2 = 3 \quad\ \ , \quad w_{21} = -6 \quad\ \ , \quad w_{22} = 0 \quad . \end{aligned} \tag{3}$$

This attractor is shown in Fig. 1b. In the following numerical simulations control is applied to this attractor. Its Lyapunov exponents have been calculated to $\Lambda_1 = 0.22$ and $\Lambda_2 = -3.3$, confirming that the attractor is chaotic. They indicate that the mean expansion is fairly weak so that nearby states separate slowly from each other on a large part of the attractor.

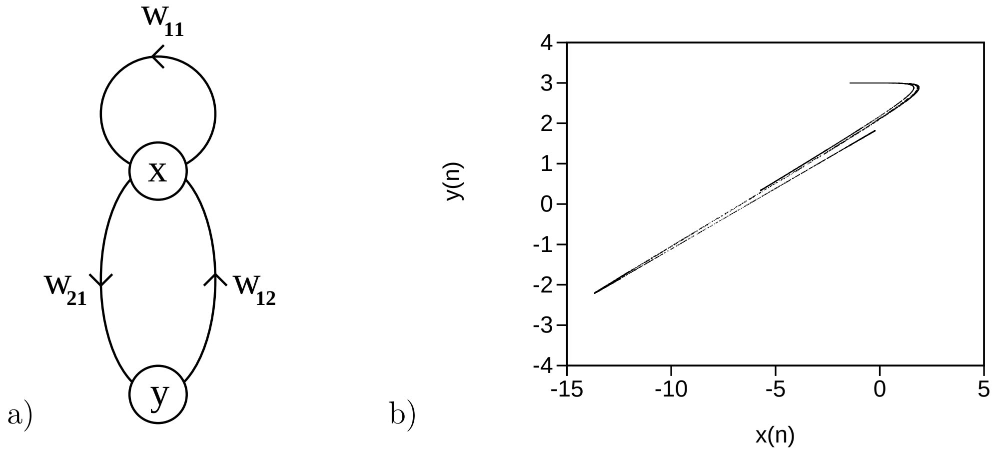

Figure 1: a) The chaotic module consisting of a self- inhibitory neuron with activity x_n, and an excitatory neuron with acitvity y_n at time step n. b) The chaotic attractor of this module for parameters as in equation (2).

The Lyapunov exponents only measure mean asymptotic expansion and contraction rates of the system. Locally in state space, expansion and contraction vary drastically. This can be read already from the Jacobi matrix Df of the system (1) given by

$$Df(\underline{x}) = \begin{pmatrix} w_{11}\sigma'(x) & w_{12}\sigma'(y) \\ w_{21}\sigma'(x) & w_{11}\sigma'(y) \end{pmatrix} .$$

Because σ' is strictly positive and symmetric with maximum at $\sigma'(0) = 0.25$, the highest expansion takes place around $\underline{x} = 0$. For parts of the present attractor (Figure 1b) lying in the region $x < -5$ only contraction occurs (Stollenwerk and Pasemann, 1996). Due to this fact it turns out that complete orbits up to period ten can be stabilized by just controlling one single point in this contracting area.

On the chaotic attractor the system visits the vicinity of infinitely many unstable periodic orbits (Procaccia, 1987). That is, there is an orbit (*dense orbit*) which approaches infinitely many unstable periodic orbits arbitrarily closely. Because of this the chaotic system cannot reach the exact position of an unstable point, even without dynamic noise. After a short time the system leaves the neighbourhood of a certain periodic orbit along the *stretching* direction and comes, because of *folding* due to the nonlinear dynamics, to the neighbourhood of another period.

The first five unstable periodic orbits along the chaotic attractor are depicted in Figure 2. In Stollenwerk and Pasemann (1996) periodic points up to period ten have been calculated using Newton's method for the parameter set (3). The following orbits were found (period/number of orbits): 1/1, 2/1, 4/1, 5/2, 6/2, 7/2, 8/3, 9/4 and 10/6. No period three orbit is observed. But for periods larger than four two or more different orbits do exist. This is due to the fact that the total number of unstable periodic points grows exponentially with period

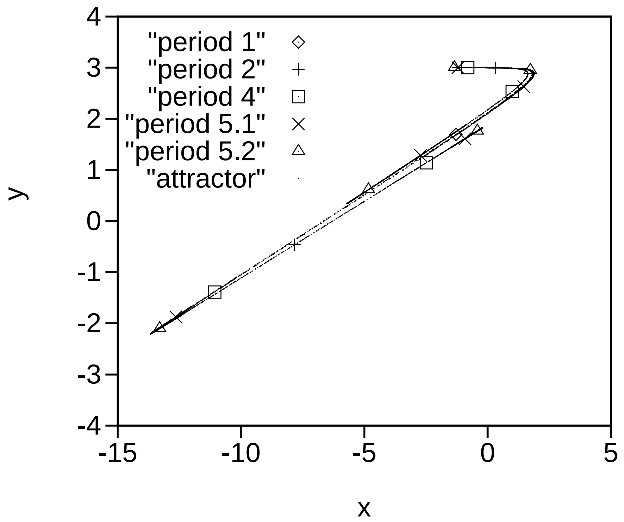

Figure 2: Periodic points of period 1, 2, 4 and of two periods 5 are shown in their position along the attractor. No period 3 exists in the system under study.

length P and rate given by the topological entropy h_T along the attractor; here $h_T = 0.422 \pm 0.004$.

3 Local Control of Unstable Periodic Orbits

The periodic orbits with points $(\underline{x}_{P,i})_{i=1}^{P}$ can be locally stabilized by varying one of the system parameters. Here we apply the control p_n to the input of the inhibitory neuron:

$$\begin{array}{lcll} x_{n+1} & = & (\vartheta_1 + p_n) & +w_{11} \cdot \sigma(x_n) + w_{12} \cdot \sigma(y_n) \quad , \\ y_{n+1} & = & \vartheta_2 & +w_{21} \cdot \sigma(x_n) \quad . \end{array} \qquad (4)$$

The control is applied only if the system comes close to a periodic point $\underline{x}_{P,i}$; i.e. if the control signal stays small

$$|p_n| < p^* \quad . \qquad (5)$$

Because the only requirement for control is to change the local expansion to contraction in the neighbourhood of distinct periodic points, appropriate control parameter changes p_n have to be determined. The general technique to do this is called pole placement, reviewed e.g. in Romeiras, et al. (1992). Besides the recently proposed and widely used Ott-Grebogi-Yorke method (OGY) (Ott, et al., 1990; Romeiras, et al., 1992) there exist alternative methods. One particularly simple and often quite effective method is the linear least squares minimization of future deviations $\Delta\underline{x}_{n+k}$. This method was applied successfully e.g. in the context of laser physics by Reyl, et al. (1993). It was compared with the algorithm of the OGY method in (Stollenwerk and Pasemann, 1996). In the following we will derive appropriate control signals, which are implementable in a neural feedforeward structure.

3.1 Instantaneous and delayed control

The dynamics of the chaotic module (1) plus instantaneous control p_n can be described as a mapping $\underline{f} : \mathbf{R}^2 \times \mathbf{R} \to \mathbf{R}^2$ given by equation (4); i.e.

$$\underline{x}_{n+1} = \underline{f}(\underline{x}_n; p_n) \quad . \tag{6}$$

In the neighbourhood of the periodic points, where

$$\Delta\underline{x}_n := \underline{x}_n - \underline{x}_{P,i}$$

is assumed to be small, we consider the linearization of the dynamics (6)

$$\mathbf{M}_i := \left.\frac{\partial \underline{f}}{\partial \underline{x}}\right|_{\underline{x}_{P,i}(0),p=0} \quad , \quad \underline{h}_i := \left.\frac{d\underline{f}}{dp}\right|_{\underline{x}_{P,i}(0),p=0} \quad .$$

At time n the deviation of the present state $\underline{x}_n$ from the periodic point $\underline{x}_{P,i}$ is taken.

In order to implement the control in a separate neural module which runs parallel to the chaotic module one has to take into account the time it needs to calculate the control. Suppose this can be done in one layer of neurons, then we have to determine a control which will influence the system one time step ahead. Hence for the dynamics

$$\begin{aligned}
x_{n+1} &= (\vartheta_1 + p_n) \quad + w_{11} \cdot \sigma(x_n) + w_{12} \cdot \sigma(y_n) \quad , \\
y_{n+1} &= \vartheta_2 \quad + w_{21} \cdot \sigma(x_n) \quad , \\
p_{n+1} &= p_{n+1}(x_n, y_n) \quad ,
\end{aligned} \tag{7}$$

the control function $p_{n+1}(x_n, y_n)$ has to be calculated. We call this type of control a *delayed control*.

In general the future development of the system depends on the whole history of the control. Taking two control steps into account the least squares condition reads

$$\|\Delta\underline{x}_{n+2}(p_n, p_{n+1})\|^2 \stackrel{!}{=} min$$

hence

$$\frac{\partial}{\partial p_n}\|\Delta\underline{x}_{n+2}\|^2 = 0 \quad , \quad \frac{\partial}{\partial p_{n+1}}\|\Delta\underline{x}_{n+2}\|^2 = 0 \quad .$$

Using the linearizations $\mathbf{M}_i$ and h_i we have

$$\begin{aligned}
\Delta \underline{x}_{n+2} &= \mathbf{M}_{i+1} \cdot \Delta \underline{x}_{n+1} + \underline{h}_{i+1} \cdot p_{n+1} + higher\ \ orders \\
&= \mathbf{M}_{i+1}\mathbf{M}_i \cdot \Delta \underline{x}_n + \mathbf{M}_{i+1}\underline{h}_i \cdot p_n + \underline{h}_{i+1} \cdot p_{n+1} \\
&= \mathbf{M}_{i+1}\mathbf{M}_i \cdot \Delta \underline{x}_n + \left(\ \underline{h}_{i+1} \ \vdots\ \mathbf{M}_{i+1}\underline{h}_i\ \right) \begin{pmatrix} p_n \\ p_{n+1} \end{pmatrix}
\end{aligned}$$

where $(\underline{h} \vdots \underline{\tilde{h}})$ denotes the (2×2)-matrix obtained from the column $\underline{h}$ by adding the column $\underline{\tilde{h}}$. Defining $\underline{d} := \mathbf{M}_{i+1}\mathbf{M}_i \cdot \Delta \underline{x}_n$, $\underline{p} := (p_n, p_{n+1})^{tr}$ and the controllability matrix $\mathbf{C}$ by

$$\mathbf{C} := \left(\ \underline{h}_{i+1} \ \vdots\ \mathbf{M}_{i+1}\underline{h}_i\ \right) \quad ,$$

evaluation of the least squares procedure yields the result

$$\underline{p} = -(\mathbf{C}^{tr}\mathbf{C})^{-1}\mathbf{C}^{tr}\underline{d} \tag{8}$$

as long as the square matrix $(\mathbf{C}^{tr}\mathbf{C})$ is invertible. Otherwise, we have to use the *Penrose pseudo inverse* $\mathbf{C}^+$ obtained by singular value decomposition (SVD) (Broomhead and Lowe, 1988) and get more generally

$$\underline{p} = -\mathbf{C}^+\underline{d} \quad .$$

Applying the result of equation (8) to the input control equation (7) we get

$$\mathbf{M}_i = \begin{pmatrix} w_{11}\ \sigma'(x_{P,i}) & w_{12}\ \sigma'(y_{P,i}) \\ w_{21}\ \sigma'(x_{P,i}) & 0 \end{pmatrix} =: \begin{pmatrix} \alpha_i & \beta_i \\ \gamma_i & 0 \end{pmatrix} \quad , \quad \underline{h}_i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad .$$

Since here $(\mathbf{C}^{tr}\mathbf{C})$ is invertible, we use equation (8) to obtain

$$\begin{pmatrix} p_{n+1} \\ p_n \end{pmatrix} = \begin{pmatrix} -\beta_{i+1}\ \gamma_i & 0 \\ -\alpha_i & -\beta_i \end{pmatrix} \cdot \Delta \underline{x}_n \quad .$$

From this result we obtain for the instantaneous control

$$p_n = \left(\ -\alpha_i\ \ ,\ \ -\beta_i\ \right) \cdot \Delta \underline{x}_n$$

as derived in Stollenwerk and Pasemann (1996) and for the delayed control

$$p_{n+1} = \left(\ -\beta_{i+1} \cdot \gamma_i\ \ ,\ \ 0\ \right) \cdot \Delta \underline{x}_n$$

as in (Stollenwerk, 1995).

4 Self-Control of the Period-2 Orbit

The delayed control just derived has the form

$$\begin{aligned} p_{n+1} &= -\beta_{i+1}\gamma_i \cdot (x_n - x_{P,i}) \\ &= -w_{12}\ \sigma'(y_{P,i+1}) \cdot w_{21}\ \sigma'(x_{P,i}) \cdot (x_n - x_{P,i}) \qquad (9) \\ &\approx -w_{12}w_{21}\ \sigma'(y_{P,i+1}) \cdot (\sigma(x_n) - \sigma(x_{P,i})) \end{aligned}$$

where the approximation $\sigma(x_n) - \sigma(x_{P,i}) \approx \sigma'(x_{P,i}) \cdot (x_n - x_{P,i})$ was used. We observe that the delayed control depends only on the x_n-component of the module activity; i.e. $p_{n+1} = p_{n+1}(x_n)$. Thus we finally define

$$p^{delay}(x_n) := -w_{12}w_{21}\ \sigma'(\vartheta_2 + w_{21}\sigma(x_{P,i})) \cdot (\sigma(x_n) - \sigma(x_{P,i})) \qquad (10)$$

or correspondingly

$$p^{delay}(x_n) =: \varphi_i\ \sigma(x_n) + \psi_i \qquad (11)$$

with coefficients

$$\varphi_i(x_{P,i}) := -w_{12}w_{21}\ \sigma'(\vartheta_2 + w_{21}\sigma(x_{P,i}))\ , \qquad (12)$$

$$\psi_i(x_{P,i} := w_{12}w_{21}\ \sigma'(\vartheta_2 + w_{21}\sigma(x_{P,i}))\sigma(x_{P,i}) \qquad (13)$$

depending only on the periodic points $x_{P,i}$.

4.1 Small control signals using cut-off functions

Now we want to apply the delayed control in terms of a local control. The corresponding smallness condition (5) of the control parameter p can be expressed for instance by a cut-off function

$$\Phi_{p^*}(p) := \begin{cases} p & for\ |p| < p^* \\ 0 & otherwise \end{cases} \quad .$$

An approximation of Φ can be realized of course by an appropriate combination of four sigmoids (2), and it is constructed here in the following way: Taking into account the symmetry $\sigma(-x) = 1 - \sigma(x)$ of the sigmoid (2) we obtain the parametrized approximating function $\Phi_{\underline{k}}$ in the form of a *weighted sum of sigmoids*

$$\Phi_{\underline{k}}(x) = k \cdot [\sigma(ax - \alpha) - \sigma(bx - \beta) - \sigma(bx + \beta) + \sigma(ax + \alpha)] \qquad (14)$$

with $\alpha := a \cdot c - d$ and $\beta := b \cdot c + e$. In the following we use a cut-off approximation with parameters

$$\underline{k}^* = (k, a, b, c, d, e)^{tr} = (1, 5, 50, 1, 3, 1)^{tr}\ .$$

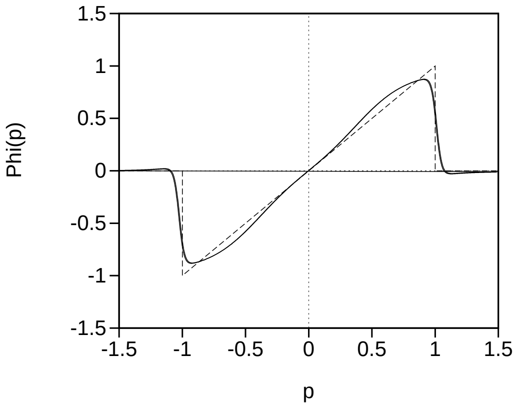

Figure 3: Approximation of the cut-off function $\Phi(p)$ (dashed line) by a combination of four sigmoids (full line).

However, for the controlling task a serious problem can occur if the cut-off approximations are insufficient near the origin. The controlled periodic orbits then might be none of the original unstable periodic orbits in the chaotic attractor; they might even lie outside the chaotic attractor. The crucial point is, that small control signals must be reproduced exactly. Thus the slope of the cut-off appoximation $\Phi_{\underline{k}}$ must be equal to 1 near the origin, i.e.

$$\left.\frac{d\Phi_{\underline{k}}(x)}{dx}\right|_{x=0} = 1 \quad .$$

This constraint gives the following condition for the parameter vector $\underline{k}$:

$$2k\ (a\ \sigma'(\alpha) - b\ \sigma'(\beta)) = 1 \quad .$$

Solving this equation with constants a, b, c, d, e, i.e. $\underline{k} = (k, 5, 50, 1, 3, 1)^{tr}$ as before, we obtain

$$k = \frac{\frac{1}{2}}{a\ \sigma'(\alpha) - b\ \sigma'(\beta)} = 0.952439 \quad .$$

The numerical value $k = 0.95$ is close to our original guess of $k = 1$ demonstrating the quality of our approximation parameters. In Figure 3 the approximating function $\Phi_{\underline{k}}$ is compared with the original cut-off function Φ_{p^*}. Especially, the exact agreement of the slope around zero is clearly visible.

Finally, we have to scale $\Phi_{\underline{k}}$ with respect to the cut-off size p^* and hence define

$$\Phi_{\underline{k}^*}(x) := p^* \cdot \Phi_{\underline{k}}(x/p^*) \tag{15}$$

with parameters $k^* := k \cdot p^*$ and $a^* := a/p^*$, $b^* := b/p^*$ in the parameter vector $\underline{k}^* = (k^*, a^*, b^*, c, d, e)^{tr}$.

By applying the approximated cut-off $\Phi_{\underline{k}^*}(x)$ to the delayed control p^{delay}, we are now able to construct a one layer network controlling the chaotic module. The composed system, having only one type of neurons, is able to stabilize an unstable periodic orbit.

4.2 Construction of consistent neural self-control

As mentioned in section 2, because the system is contracting in a large phase space domain a whole unstable periodic orbit can be stabilized by applying the control only in the neighborhood of one single selected periodic point of the orbit (Stollenwerk and Pasemann, 1995). Since the constants in the delayed control p^{delay}, given by equation (10), depend on the choice of this specific periodic point $\underline{x}_{P,i_s}$ the corresponding local control is realized by the combination of the cut-off $\Phi_{\underline{k}^*}$ (15) with p^{delay}. Because of the delayed control, selected periodic points must be mapped in the next time step to points in the most contracting part of the attractor (compare section 2). For the unstable period-2 orbit, for example, the selected point is $\underline{x}_{P=2,i_s} = (0.3107,\ 2.9976)$.

Hence, the local control signal $p_{n+1} = \Phi_{\underline{k}^*}(p^{delay}(\underline{x}_n))$ for a selected periodic point is generated by the four neurons of the control layer. Let z_μ , $\mu = 1, 2, 3, 4$ denote their activities. According to equation (10) their inputs are given by the weighted output $u_\mu \sigma(x)$ of the inhibitory neuron of the chaotic module. Their outputs $\sigma(z_n)$ project back on the inhibitory neuron via the connecting weights v_μ, $\mu = 1 \ldots, 4$. So the control module is realized as a one-layer feedforward network as shown in Figure 4.

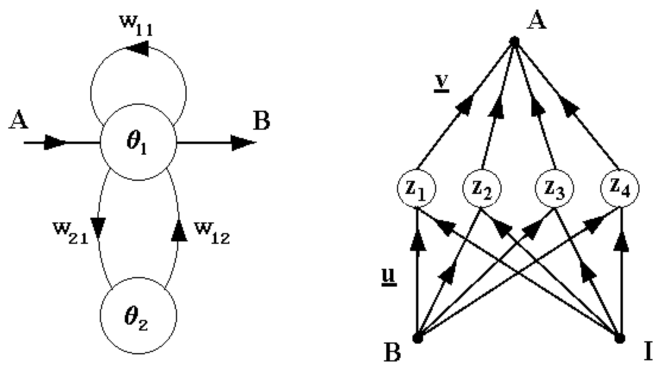

Figure 4: Chaotic module and control module with their interfaces A and B; I denotes the inhibiting inputs for the control neurons.

Now the parameters $\underline{\theta}$, $\underline{u}$ and $\underline{v}$ of the control network, with θ_μ denoting the corresponding bias terms, can be expressed in terms of the parameters φ_i and ψ_i of the delayed control given by equations (12) and (13), and $\underline{k}^*$ characterizing the approximated cut-off function (15), i.e.

$$\begin{array}{lll} u_1 = a^*\varphi_i & \theta_1 = a^*\psi_i - \alpha & v_1 = k^* \\ u_2 = b^*\varphi_i & \theta_2 = b^*\psi_i - \beta & v_2 = -k^* \\ u_3 = b^*\varphi_i & \theta_3 = b^*\psi_i + \beta & v_3 = -k^* \\ u_4 = a^*\varphi_i & \theta_4 = a^*\psi_i + \alpha & v_4 = k^* \quad . \end{array}$$

Using this construction (Figure 4) we call the composition of the chaotic module with a corresponding control module a *self-controlling* chaotic neural network. This network is consistent in the sense that chaotic dynamics as well as its control is realized in one and the same system using the same type of neurons everywhere. The effect of this controlling scheme is demonstrated in Figure 5 for the unstable period-2 orbit using a cut-off size $p^* := 0.05$. In Figure 5a the resulting control signal $\Phi(p(x))$ is depicted in a schematic way as a function of the activity x of the inhibitory neuron. This function has a nonvanishing part only around the selected periodic point. Its exact shape is of course the one depicted in Figure 3, but with larger slope because of the distorsion by $p^{delay}(x)$. The time series shown in Figure 5b demonstrates the successful control after a transient motion along the chaotic attractor of Figure 1b.

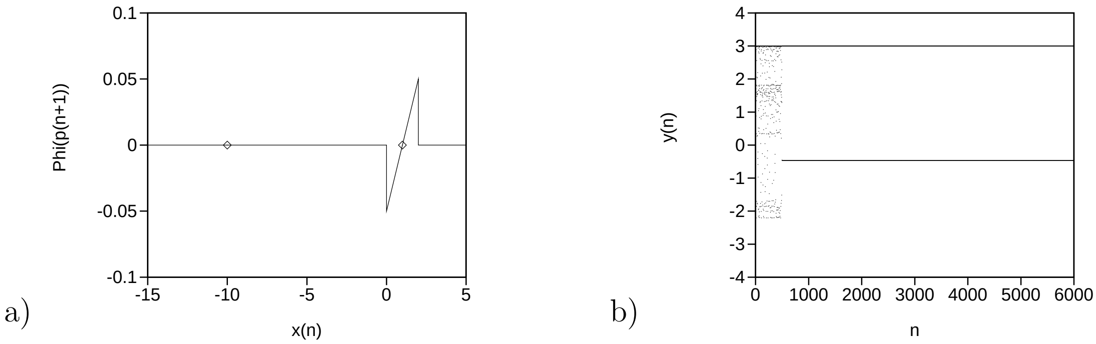

Figure 5: Self-control for the period-2 orbit: In a) the control function is depicted in a schematic way while in b) the time series shows successful control after a transient before the local control signal becomes effective.

For the period-4 and both period-5 orbits local control around selected periodic points is equally efficient as for the period-2 case. For the period-4 orbit a convenient periodic point is given by $\underline{x}_{P=4,i_s} = (1.0010,\ 2.5359)$. Selected periodic points for the two period-5 orbits are $\underline{x}_{P=5.1,i_s} = (1.4625,\ 2.6293)$ and $\underline{x}_{P=5.2,i_s} = (1.7355,\ 2.9525)$. In agreement with our theoretical considerations simulations show that the application of our local self-control to other than these selected points does not work as effectively or even fails completely.

5 Two Possibilities for Period Switching

Having verified that unstable orbits of period 2, 4 and 5, for example, can be stabilized by local control in one periodic point, we now investigate two distinct ways of switching between these different orbits. For that purpose we introduce inhibiting inputs to all neurons of a control module. In order to stabilize N different unstable periodic orbits we now have to introduce N different controllers with different local controlling areas corresponding to the N selected periodic points.

Inhibiting all of these controllers should of course leave the system on the chaotic attractor. Selective disinhibition of one controller should then stabilize the corresponding orbit after some transient time. A second situation occurs if all controllers are active. We will distinguish these two cases denoting them as *selective switching* by external control inputs and *spontaneous switching* mediated by internal or external noise of the chaotic module.

For simulations we use a setup with three control modules denoted by $C(2)$, $C(4)$, and $C(5.1)$. They stabilize the orbits 2, 4 and 5.1 with periods two, four and five, respectively. Each controller is of the form depicted in Figure 4. The corresponding local control areas do not overlap for the three different periods if $p^* = 0.05$. In general, areas of nonvanishing control scale of course in height as well as in width by p^*. The second method of switching (spontaneous switching) will use the fact of nonoverlapping control regions for stabilizing different orbits in a random sequence induced by dynamical noise.

5.1 Deterministic switching by external inputs

Because the transfer function σ satisfies $\sigma(x) \approx 0$ for large negative x, inhibiting all neurons of a control module $C(\nu)$ by a strong negative input I^ν will set the corresponding control signal for orbit ν to zero. Thus, inhibiting all but one controller at a time will stabilize the corresponding periodic orbit. Switching to another orbit is then performed as follows: First the active controller is also inhibited and the corresponding orbit will be destabilized again. Then, liberating a different controller will stabilize - after a transient along the chaotic attractor - the new periodic orbit. In this sense the chaotic attractor links all possible unstable periodic orbits and its dynamics might be interpreted as an *attentive state* of the kind suggested for instance by Skarda and Freeman (1987) and Ding and Kelso (1991).

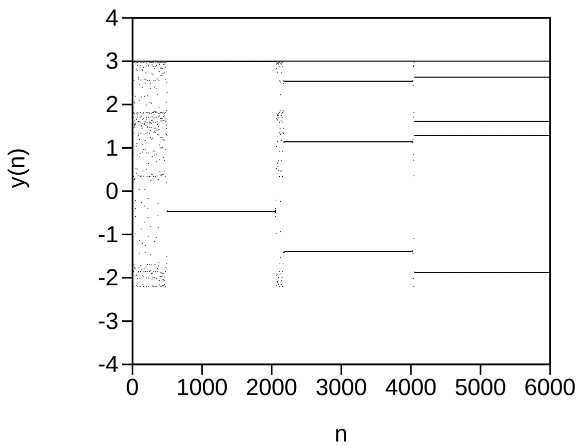

Figure 6: Deterministic switching by varying control inputs from period 2 to period 4 after 2000 time steps and finally to period 5.1 after 4000 steps.

In Figure 6 selective switching is demonstrated by simulation results with the described setup: For time steps $n = 1$ to $n = 2000$ the period two controller $C(2)$ is activated. After a transient of about 500 time steps the period two orbit is in fact stabilized. Then, for $n = 2000$ to $n = 4000$ only the period four controller $C(4)$ is de-inhibited and successfully stabilizes the orbit after a short transient. Finally, after $n = 4000$ the periodic orbit 5.1 is stabilized with controller $C(5.1)$. The inhibition for all neurons of a controller $C(\nu)$ must be sufficiently large so that the controller does not perturb the original chaotic dynamics when all controllers are turned off. For simulations the inhibition was given by a constant $I^{\nu}_{\mu} = -10,000$, $\mu = 1, ..., 4$. For controlling a specific orbit ν controller $C(\nu)$ is liberated by setting the corresponding inhibiting inputs I^{ν}_{μ} to zero.

5.2 Spontaneous switching by dynamic noise

When activating all three controllers at a time, i.e. all external control inputs I^{ν}_{μ} are set to zero, the system will, after a transient on the chaotic attractor, enter one control area in phase space, being controlled there forever. Which periodic orbit will be stabilized depends on the initial conditions and the relative size of each control region in respect to the invariant measure of the whole attractor.

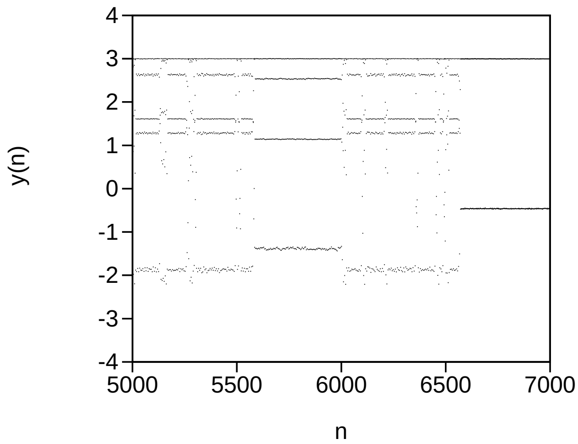

Figure 7: Spontaneous switching by internal dynamic noise between all three deterministically controlled periods 5.1, 4, and finally 2. In between randomly occuring escapes to transients along the chaotic attractor are visible.

But if an internal (or external) dynamic noise term $\underline{\xi}_n \in \mathbf{R}^2$ is introduced into the chaotic module (1), the noisy system can escape from a once controlled orbit and move along the attractor until being eventually captured in another control area. This new periodic orbit will be destabilized again by the noise, and so on.

Simulations reveal, that the simplest type of noise, i.e. Gaussian white noise with constant variance, is sufficient to induce spontaneous switching between orbits of period two, four and five. For the following example we applied noise with a standard deviation of $\sigma = 0.002$ to the inputs of the chaotic module. This corresponds to 0.04% of the standard deviation for the x-signal, and to 0.1% of the standard deviation for the y-signal. Part of the resulting time series is depicted in Figure 7 where all three orbits 2, 4 and 5.1 are visited. Although each period is controlled only locally around a single periodic point, the period five orbit is stabilized most often, eventually destabilized by the noise and revisited after a short transient. At around time $n = 5600$, however, the period-4 orbit is met after a transition and can be stabilized for about 400 time steps before becomming unstable again. After these switches the system even locks in to the period-2 orbit at around time step 6600.

Also switching back and forth between period two and period five can be detected, however, with a larger amount of noise. In our simulations the duration of continuous stabilization of the orbits is distributed quite unevenly. The reason for this is, that we restricted ourselves to a constant cut-off $p^* = 0.05$ for all control areas. A more balanced duration of the different periodic orbits can be achieved by adjusting the cut-offs $p^{*\nu}$ with respect to the attractor's density around the selected control points $\underline{x}_{P,i_s}$.

6 Summary and Discussion

In this article we have shown, that in principle a consistent self-controlling chaotic neural systems is realizable. That is, a modular neural network composed of a chaotic system and a controller, which is able to switch between different oscillatory modes. Switching will occur as response to external control signals or as an internal property of a noisy system.

Starting with the delayed algorithmic control for a given unstable periodic orbit, all bias terms and weights of the corresponding control module can be calculated. So no learning procedure is applied here. With multi-cut-offs generating the local control around every periodic point of an orbit, the problem scales of course with the number of periodic points. Hence, the size of such a controller will grow exponentially with increasing period length, the growth rate being roughly the topological entropy.

One possible way out of the scaling problem is the "one point control" as developed in (Stollenwerk and Pasemann, 1996). Here we have shown, that its delayed version (Stollenwerk, 1995) can be implemented in one layer feedforeward networks, and that switching between different periodic orbits can be achieved by this control technique. The combination of the chaotic module with a number of different controllers can then serve as a special purpose system. For instance as a "chaotic categorizer" of the type described in Babloyantz and Lourenço (1994). Or it it can provide a dynamic short term memory, where information is coded by different periodic orbits. In contrast to the usual content addressable memories, information is not stored in competitive coexisting attractors here, but a selective control mechanism is used, which transforms one oscillating mode of the composed system into a different one. For a noisy chaotic module and all controllers active at the same time, spontaneous switching between different periodic orbits may be functionally related to a random search memory.

References

[1] Alsing, P.M., Gavrielides, A., and Kovanis, V. (1994). Using neural networks for controlling chaos. *Physical Review E*, **49**, 1225–1231.

[2] Babloyantz, A., and Destexhe, A. (1986). Low-dimensional chaos in an instance of epilepsy. *Proceedings of the National Acadademy of Sciences USA*, **83**, 3513–3517.

[3] Babloyantz, A., and Lourenço, C. (1994). Computation with chaos: A paradigm for cortical activity. *Proceedings of the National Acadademy of Sciences USA*, **91**, 9027–9031.

[4] Babloyantz, A., Salazar, J.M., and Nicolis, C. (1985). Evidence of chaotic dynamics of brain activity during the sleep cycle. *Physics Letters*, **A111**, 152–156.

[5] Broomhead, D., and Lowe, D. (1988). Multivariable function interpolation and adaptive networks. *Complex Systems*, **2**, 321–355.

[6] Der, R., and Herrmann, M. (1994). Q-learning chaos controller. *Proceedings IEEE Conference on Neural Networks*, (Orlando, Florida), 2472–2475.

[7] Ding, M., and Kelso, S. (1991). Controlling chaos: A selection mechanism for neural information processing. In D. W. Duke, W. S. Pritchard (Eds.), *Measuring Chaos in the Human Brain* (pp. 17–31). Singapore: World Scientific.

[8] Dressler, U. and Nitsche, G. (1992). Controlling chaos using time delay coordinates. *Phys. Rev. Lett.*, **68**, 1 – 4.

[9] Elbert, T., Ray, W.J., Kowalik, Z.J., Skinner, J.E., Graf, K.E., and Birbaumer, N. (1994). Chaos and physiology: Deterministic chaos in excitable cell assemblies. *Physiological Review, American Physiological Society*, **74**, 1–47.

[10] Freeman, W.J. (1988). Strange attractors that govern mammalian brain dynamics shown by trajectories of electroencephalographic (EEG) potentials. *IEEE Transactions On Circuits and Systems*, **35**, 781–783.

[11] Freeman, W.J. (1992). Tutorial on neurobiology: From single neurons to brain chaos. *Internationl Journal of Bifurcation and Chaos*, **2**, 451–482.

[12] Freeman, W.J. (1993). The emergence of chaotic dynamics as a basis for comprehending intentionality in experimental subjects. In K.h. Pribram (Ed.) *Rethinking Neural Networks*, (pp. 507–514). New Jersey: Hillsdale.

[13] Freeman W.J., and Barrie, J.M. (1994). Chaotic oscillations and the genesis of meaning in cerebral cortex. In G. Buzsáki, R. Llinás, W. Singer, A. Berthoz and Y. Christen (Eds.), . *Temporal Coding in the Brain* (pp. 13–37). Berlin: Springer-Verlag.

[14] Guevara, M.R., Glass, L., Mackay, M.C., and Shrier, A. (1983). Chaos in neurobiology. *IEEE Transactions on Systems, Man, and Cybernetics*, **13**, 790–798.

[15] Hayashi, H., & Ishizuka, S. (1995). Chaotic responses of the hippocampal CA3 region to a mossy fiber stimulation in vitro. *Brain Research*, **686**, 194–206.

[16] Kapitaniak, T. (1996). *Controlling Chaos - Theoretical and Practical Methods in Non-linear Dynamics.* San Diego: Academic Press.

[17] Lourenço, C., and Babloyantz, A. (1994). Control of chaos in networks with delay: A model for synchronization of cortical tissue. *Neural Computation*, **6**, 1141–1154.

[18] Ott, E., Grebogi, C., and Yorke, J.A. (1990). Controlling chaos. *Physics Review Letters*, **64**, 1196–1199.

[19] Pasemann, F. (1995). Neuromodules: A dynamical systems approach to brain modelling. In H. Herrmann, E. Pöppel, D. Wolf (Eds.), *Supercomputing in Brain Research - From Tomography to Neural Networks*, (pp. 331–347). Singapore: World Scientific.

[20] Procaccia, I. (1987). Exploring deterministic chaos via unstable periodic orbits. *Nuclear Physics* **B**, *Proceedings Supplement*, **2**, 527–538.

[21] Reyl, C., Flepp, L., Badii, R., and Brun, E. (1993). Control of NMR-laser chaos in high-dimensional embedding space. *Physics Review E*, **47**, 267–272.

[22] Romeiras, F.J., Grebogi, C., Ott, E., and Dayawansa, W.P. (1992). Controlling chaotic dynamical systems. *Physica D*, **58**, 165–192.

[23] Schiff, S.J., Jerger, K., Doung, D.H., Chang, T., Spano, M.L. and Ditto, W.L. (1994). Controlling chaos in the brain. *Nature*, **370**, 615–620.

[24] Sepulchre, J.A. and Babloyantz, A. (1993). Controlling chaos in a network of oscillators. *Physics Review E*, **48**, 945–950.

[25] Skarda, C.A., and Freeman W.J. (1987). How brains make chaos in order to make sense of the world. *Behavioural Brain Science*, **10**, 161–195.

[26] Stollenwerk, N. (1995). Self-controlling chaos in neuromodules. In H. Herrmann, E. Pöppel, D. Wolf (Eds.), *Supercomputing in Brain Research - From Tomography to Neural Networks*, (pp. 421–426). Singapore: World Scientific.

[27] Stollenwerk, N. and Pasemann, F. (1996). Control Strategies for Chaotic Neuromodules. *International Journal of Bifurcation and Chaos*, **6**, 633–703.